Policy Analysis with Endogenous Migration Decisions: The Case of Left-Behind Children in China

Rebecca Myerson*

April 5, 2017

Abstract
I present a model of parental decision-making with endogenous migration to study the human capital of children in migrant families, focusing on the case of rural China. I derive two main results related to (i) the effect of parental migration on children’s human capital, and (ii) the effects of government policies that increase services for children. First, I show that if government spending is a substitute for parental spending and parent time with the child is weakly complementary to spending on the child, then children’s human capital decreases as they become left-behind by migrant parents. This sheds light on a puzzle in the empirical literature about the ambiguous effects of parental migration on the human capital of left-behind children. Second, in some cases, government policies that increase services for rural children have the unintended consequence of separating children from parents due to endogenous migration effects, thereby reducing child human capital. More broadly, the analysis demonstrates how focusing on marginal treatment effects can facilitate signing comparative statics that are otherwise difficult to sign.

Key words: China; left-behind children; marginal treatment effects; policy analysis; migration; household behavior. JEL Codes: O15, D13, R23, R50, H75

*Contact information: Rebecca Myerson, Assistant Professor, University of Southern California School of Pharmacy Department of Pharmaceutical and Health Economics, rmyerson@usc.edu, 635 Downey Way, VPD Office 414E, Los Angeles, CA 90089, (213)-821-8872. I am grateful for feedback and suggestions from Pietro Biroli, Steven Durlauf, Matthew Kahn, Sara Moreira, Roger Myerson, Jesse Naidoo, Roberto Robatto, Bradley Setzler, Dali Yang, and participants of the East Asia Workshop and Center for Program Evaluation workshops at the University of Chicago, the Population Association of America conference, the Michigan Development Day conference, and the Association for Public Policy and Management conference. I declare no conflicts of interest.
1 Introduction

The human capital of children in migrant families is a topic of growing concern. China provides a case study with wider applicability because migrants within China lose access to some government social services, as many international migrants do. Inequality in economic opportunity by geographic location spurred what has been called the largest rural-urban labor migration in human history [Roberts, 1997]. The number of rural workers in China’s urban areas reached an estimated 277 million in 2015, accounting for about 36% of the total workforce [National Bureau of Statistics of People’s Republic of China, 2015]. Due to the Chinese residential registration permit system (the *hukou* system), access to health and education services is more limited for individuals not living in their official place of residence [Cheng and Selden, 1994, Lai et al., 2009, Wang, 2007, Wei, 2000, Wu and Treiman, 2004, Ye et al., 2005]. As a result, many children remain in rural areas while one or both parents migrate for work. In 2010, there were an estimated 61 million so-called “left-behind” children in rural China who lived apart from one or more migrant parents, equivalent to 38% of all rural children in China [All China Women’s Federation, 2008, 2013, China Youth Research Center, 2006].

Establishing whether being left-behind by economic migrant parents is good or bad for children’s human capital is important for shaping public policy, but the extensive empirical research on this question has shown mixed results [Fan et al., 2010, Gao et al., 2010, He et al., 2012, Hu et al., 2014, Huang et al., 2015, Jia and Tian, 2010, Liu et al., 2010, Lu, 2011, 2012a,b, Luo et al., 2008, Wen et al., 2015, Xu and Xie, 2013, Ye et al., 2005, Ye and Pan, 2011, Zhao et al., 2014, Zhou et al., 2014, 2015]. However, Antman [2012] and others argue that mixed results are to be expected: left-behind children experience both positive factors (parental remittances) and negative factors (parental absence), and therefore the impact of being left behind could be negative or positive depending on the balance of these two factors and their relative importance for developing any given capability. As a result, the empirical literature faces significant challenges in identifying the total impact of being left-behind by migrant parents on rural children’s human capital.

This paper provides a new answer to this question by using economic theory to isolate a case where the net impact of being left-behind on children can be made clear. The first main result of this paper is as follows: if government money is a substitute for parental spending and parent time with the child is weakly complementary to spending on the child, then becoming left-behind reduces children’s human capital. While not a universal case, the
case where government spending is a substitute for parental spending is likely an important one. This possibility has been cited as an explanation for low measured impacts of some programs that increase government spending on children. (See Becker [1981], pages 125-126 and 251-253, as well as Del Boca et al., 2014, 2016.)

This result is derived using a model wherein parents are the sole decision-makers and care about their own consumption and their children’s human capital. Parents can choose between multiple migration scenarios (migrating and bringing their children, migrating and leaving their children behind, or not migrating), and they can choose their levels of time and monetary investments in their children. To derive the finding that becoming left-behind reduces children’s human capital, I exploit the fact that children are left-behind by migrant parents at the moment when their parents cross a margin of indifference between two migration scenarios. I focus on parents at this margin of indifference and use the margin of indifference as an additional equation to sign the otherwise difficult-to-sign comparative static.

A corollary of this result is that in some cases, government policies that increase services for rural children have endogenous migration effects that separate parents from children, thereby reducing the benefits of the policy for children. To develop this argument, I first observe that if parents were previously indifferent between migrating with their child vs. without their child, an increase in generosity of location-specific government services would determine whether the child is left-behind. I then unpack the effects, including migration effects, of two broad classes of policy changes designed to help children from rural areas. The first class of policy changes increase government spending on rural non-migrant children, and the second class of policy changes increases government spending on rural migrant children.

Both types of policies increase government spending, which directly benefits children, but I show that migration effects can augment or undermine these benefits. In particular, when children become left-behind by migrant parents due to an increase in government spending in rural areas, this decreases their human capital. (This holds both for children who previously lived in rural areas with their parents, and for children who previously lived in urban areas with their parents as rural-to-urban migrants.) Likewise, when previously left-behind children rejoin their migrant parents due to an increase in government spending for migrant children, this increases the children’s human capital.

This paper contributes to the literature on migration, human capital, and policy analysis in multiple ways. First, by identifying conditions under which the relationship between
human capital and becoming left-behind by migrant parents can be made clear, my findings shed light on a puzzle in the empirical literature about the effects of parental migration on children. The paper also contributes to the “welfare magnet” literature which considers endogenous migration effects as an important part of policy analysis [Allard and Danziger, 2008, Berry et al., 2003, Borjas, 1999, Levine and Zimmerman, 1999, Schwartz and Summers, 2014]. In particular, my findings imply that welfare magnet migration effects can undermine the effects of a location-specific policy if the people making decisions about migration (in my model, parents) are not the intended beneficiaries of the policy (in my model, children). This insight contributes to discussions about the optimal design of government policies.

The model and analysis in this paper also contribute to the theoretical literature. The model integrates elements of classic models of labor migration with elements of classic models of parental investment in children, in an effort to capture the key trade-offs involved in parental economic migration [Becker and Tomes, 1979, Harris and Todaro, 1970, Solon, 2004]. In addition, the analysis demonstrates a new approach to signing difficult-to-sign comparative statics in policy analysis: focusing on the impact of taking a certain path of action for individuals at the margin of doing so, also known as a marginal treatment effect [Carneiro et al., 2010, Heckman and Vytlacil, 1999, 2001, 2007, Heckman, 2010]. In particular, I use parents’ margin of indifference as an additional equation to sign a comparative static that is otherwise difficult to sign, namely, the effect of becoming left-behind on a child’s human capital.

The remainder of the paper proceeds as follows. Section 2 briefly summarizes the empirical motivation for the theoretical framework, focusing on the case of China. Section 3 presents the model and Section 4 presents the results. Section 5 demonstrates applications of the findings for policy analysis. Section 6 discusses these results in the context of the theoretical literature.

2 Empirical motivation

2.1 Location-specific government investment in children

This paper focuses on the example of China, where geographic variation in government spending on children is formalized through the hukou system. Although the hukou system is particular to China, lessons drawn from this context could be relevant for government
policies in other contexts that use place-specific government benefits as incentives to reduce urban crowding, stem rural-to-urban migration, or attract a particular type of resident to a community [Oberai, 1983, Simmons, 1981, Tiebout, 1956, Todaro, 1969b].

Under the hukou system, social services provided by the Chinese government follow a pattern wherein official residents of a given location (i.e., people with local hukou) are eligible for more government-provided services than people who are not official residents of that location [Cheng and Selden, 1994, Lai et al., 2009, Wang, 2007, Chan and Buckingham, 2008]. Despite hukou reforms in 2014, it is expected that availability of hukou in many urban areas, and thereby government services for rural children in urban areas, will remain limited. Therefore, the pattern wherein rural people receive lower levels of government services if they migrate to urban areas should be taken into account in modeling the response of the Chinese family to changes in government services.

2.2 Mixed empirical evidence on the impact of being left-behind on children’s human capital development

Antman, 2012, Park and Lee, 2010, Wen et al., 2015, Zhou et al., 2014 and others argue that because left-behind children experience both positive factors (increased parental spending on children via remittances) and negative factors (decreased parental time with children), the total impact of being left-behind on children’s development could be negative or positive, depending on the balance of these factors. Below, I cite examples from the empirical literature on the impact of being left-behind on children in China with an eye to the potential impacts of increased remittances and lost time with parents.

Although remittances support children’s education, these funds are garnered at the cost

---

1In the reforms, policy-makers set a goal of granting hukou from urban locations to 100 million people who currently hold hukou from rural locations. These additional hukou were to be allocated based on size of the destination city: small cities should be “fully open,” mid-sized cities should be opened in an “orderly” fashion, and hukou in the largest cities should remain “strictly controlled” [Zhang et al., 2016]. As such, the reforms did not change two characteristics of the existing system: (1) hukou in more desirable cities that are larger and provide more generous benefits are available only to those who meet criteria set by the city (often, a points system based on factors like wealth, occupation, and education), and (2) individuals who forgo their rural hukou forgo the land entitled to them in their home village [Solinger and Hu, 2012]. For many rural people, obtaining one of the urban hukou that are within their reach may continue to not be worth the cost, a prediction borne out by earlier hukou reforms [Tiezzi, 2015, Wall Street Journal, 2014, Wang, 2007, Chan and Buckingham, 2008].

2Remittances can account for a third of migrants’ labor earnings in China and can increase the consumption of rural households by 20 percent, with the amount of remittances varying based on the needs of family members left behind [De Brauw and Rozelle, 2008, Du et al., 2005, Liu and Reilly, 2006, Taylor et al., 2003].
of parental time with children, which is also an important determinant of academic performance [Brown, 2006, Hu, 2012, Knight et al., 2009, Wen et al., 2015]. Loss of parental time can also increase the need for children’s labor at home, which may interfere with studying [Chang et al., 2011, McKenzie and Rapoport, 2006, Ye and Pan, 2011]. Accordingly, the observed impact of being left-behind by migrant parents on children’s school attendance and academic skill development has been mixed [Chen et al., 2009, Lee and Park, 2010, Wang, 2014, Zhou et al., 2014, Zhu et al., 2002].

A similarly mixed effect has been shown for left-behind children’s health. Research on the impact of being left-behind on children’s nutrition has shown that remittances support the purchase of nutritious food such as meat for left-behind children, while the lack of an educated caregiver is a risk factor for anemia and underweight [DeBrauw and Mu, 2011, Graham and Jordan, 2013, Lu, 2012b, Luo et al., 2011, Zhou et al., 2015]. The loss of parental time seems to be detrimental to children’s mental health, with higher rates of psychological distress observed among children who were left-behind at an earlier age [Cheng and Sun, 2015, Fan et al., 2010, Graham and Jordan, 2011, Jia and Tian, 2010, Liu et al., 2009, Wu et al., 2015]. Lack of parental supervision and parental remittances may also contribute to higher rates of unhealthy behaviors such as smoking and drinking among left-behind adolescents [Gao et al., 2010, Huang et al., 2015].

2.3 Implications of the empirical evidence for the theoretical model

The theoretical model in this paper is informed by two patterns in the empirical literature. First, parental spending on children and parental time with children are differently impacted by migration and are both important for children’s human capital development. Accordingly, I track both factors in the model and allow parents’ migration decisions to affect children through both channels. Second, parental spending on children and parental time with children impact different child outcomes in different ways. This raises concerns about how to aggregate the mixed impacts of being left-behind on these various domains to address whether being left-behind is good or bad for children’s human capital overall. To side-step this issue, I use a single summary metric of child human capital in the model, as is done in the theoretical literature on intergenerational mobility, rather than modeling the impacts on each domain separately and aggregating the results [Becker et al., 2015, Becker and Tomes, 1979, Loury, 1981, Solon, 2004].
3 Model

The model is constructed to be as simple and general as possible while incorporating the key factors that affect child human capital and parental migration: parental time and money inputs to the child, geographic differences in parental wages, and government programs supporting the child.

I consider a household that contains one parent and one child, both with rural *hukou*. The model can be extended to include multiple parents or grandparents without changing the results, as shown in Appendix B. See Figure 1 for a summary of the model including policy inputs, parental decisions, and consequences for parents and children.
3.1 Utility, budget constraint, and child human capital

Following previous literature (Becker et al. 2015, Becker and Tomes 1979, Loury 1981), parents are the sole decision-makers in the household and maximize a smooth function of their own consumption \( C \) and their child’s human capital \( h \):

\[
U = u(C, h)
\]

Parental migration and child migration are denoted as \( M^p \) and \( M^c \), respectively. \( M^p = 1 \) if the parent migrates and \( M^p = 0 \) if the parent does not migrate. Likewise, \( M^c = 1 \) if the child migrates and \( M^c = 0 \) if the child does not migrate.

Parents’ wages are denoted as \( w \), where:

\[
w = \begin{cases} 
  w_u & \text{if parent migrates (i.e., } M^p = 1) \\
  w_r & \text{if parent doesn’t migrate (i.e., } M^p = 0) 
\end{cases}
\]

As in Harris and Todaro [1970], wages are not altered by individual agents’ choices.

Each parent is endowed with one unit of time. When the child and parent live together, the parent spends some fraction of their time \( \tau \) with the child; parents work the remainder of the time. In addition, parents have the option of investing money (of amount \( I \)) in their children. Therefore, parents face the budget constraint:

\[
C + I \leq \begin{cases} 
  w & \text{if parent migrates without the child (i.e., } M^p \neq M^c) \\
  w(1 - \tau) & \text{if parent and child live in the same location (i.e., } M^p = M^c) 
\end{cases}
\]

Finally, the child’s human capital is determined by the amount of time spent with the parent \( \tau \), and monetary resources spent on him or her. Spending on the child comes from the parents \( I \) and from the government \( G \) and both increase child human capital: \( h_I > 0 \) and \( h_G > 0 \). Child human capital can therefore be written as \( h(I, G, \tau) \), where government spending on the child \( G \) varies based on the child’s migration status:

\[
G = \begin{cases} 
  G_u & \text{if child migrates (i.e., } M^c = 1) \\
  G_r & \text{if child doesn’t migrate (i.e., } M^c = 0) 
\end{cases}
\]
3.2 Assumptions

I impose some conditions on the utility function, human capital production function, and government policies and wages. Most of these are standard in the literature. I provide additional details below.

**Assumption 1** Parents care about their child’s human capital and their own consumption, but with decreasing marginal returns to each.

\[ u_h(C,h) > 0; \quad u_C(C,h) > 0 \]

\[ u_{hh}(C,h) < 0; \quad u_{CC}(C,h) < 0 \]

In addition, Inada conditions hold for both C and h.

**Assumption 2** \( u(C,h) \) is additively separable in C and h:

\[ u(C,h) = \mu(C) + \nu(h) \]

See Becker et al. [2015] and others for examples of this assumption deployed in the theoretical literature.

**Assumption 3** Time and money investments in children increase child human capital, but there are decreasing marginal returns to each. Furthermore, I and G are substitutes:

\[ h_I(I,G,\tau) > 0; \quad h_G(I,G,\tau) > 0; \quad h_{\tau}(I,G,\tau) > 0 \]

\[ h_{II}(I,G,\tau) < 0; \quad h_{GG}(I,G,\tau) < 0; \quad h_{\tau\tau}(I,G,\tau) < 0 \]

\[ h_{IG}(I,G,\tau) < 0 \]

The possibility that “government programs may have substitution effects by changing rates of return on parental investments in children,” has been noted since the first joint Becker-Tomes paper on intergenerational mobility. See Becker and Tomes [1994], p. 273-74, for a discussion.
**Assumption 4** Parental time weakly increases the productivity of parental monetary investments in the child:

\[ h_\tau \geq 0 \]

For example, the books or educational toys may have more impact if the parent spends time teaching the children how to use them. Likewise, parental time with the child may have larger impact if the parent has purchased educational books or toys that they can use together.

**Assumption 5** The government spends more money on children who live in their official place of residence:

\[ G_r > G_u \]

As noted in section 2, the \( G_r > G_u \) assumption is consistent with the hukou system in China, including recent reforms.

**Assumption 6** Wages for parents are higher in the urban area than in the rural area:

\[ w_r < w_u \]

This assumption is broadly consistent with the scale of rural-to-urban labor migration in China [National Bureau of Statistics of People’s Republic of China, 2015].

### 3.3 Parental decisions

#### 3.3.1 Migration decisions

Given the above framework, each parent is faced with the following discrete choice problem:

- \( M^p = 1 \) and \( M^c = 1 \) (parent and child migrate together)
- \( M^p = 0 \) and \( M^c = 0 \) (neither parent nor child migrates)
- \( M^p = 1 \) and \( M^c = 0 \) (parent migrates and child is left behind)

Within each of these scenarios, the parent can select the optimal \( I, C, \) and perhaps \( \tau \), to maximize her utility. (When the parent and child live in different locations, \( \tau \) is set to 0.)

To define notation, let \( C^{*}_{10} \) and \( h \left( I^{*}_{10}, G_r, 0 \right) \) denote the parent’s optimal \( C \) and \( h \) conditional on \( M^p = 1, M^c = 0 \) (that is, the first subscript in \( C^{*}_{10} \) indicates \( M^p = 1 \) and second indicates \( M^c = 0 \)). Then \( U^{*}_{10} = u \left( C^{*}_{10}, h \left( I^{*}_{10}, G_r, 0 \right) \right) \) is the utility of the parent at the
optimum conditional on $M^p = 1$, $M^c = 0$. The optimal choices and utility for the other migration scenarios are similarly denoted using stars and subscripts. Using this notation, parental choices and the corresponding utility can be summarized as follows:

$$U = \max_{M^p, M^c} \left\{ \begin{array}{l} U^*_{11} = u(C^*_{11}, h(I^*_{11}, G_u, \tau^*_{11})) \\ U^*_{00} = u(C^*_{00}, h(I^*_{00}, G_r, \tau^*_{00})) \\ U^*_{10} = u(C^*_{10}, h(I^*_{10}, G_r, 0)) \end{array} \right. $$

Any of the three migration scenarios could be most appealing to parents, and the key tradeoffs are as follows. First, children receive more government services in rural areas by Assumption 5. Second, parents have higher wages in urban areas than rural areas by Assumption 6. Third, migrating parents are unavailable to spend time with left-behind children. If the importance of parental time is sufficiently large then migrating parents will not wish to leave their children behind.

### 3.3.2 Optimal decisions if the child and parent live in the same location

When a child and parent live in the same location ($M^p = M^c$), the parent has the option of forgoing some income to spend time with the child. In this case, the maximization problem is:

$$\max_{I, \tau, C} \{ u(C, h(I, G, \tau)) + \lambda (w(1 - \tau) - C - I) \}$$

where $\lambda$ is the Lagrange multiplier. The first order conditions are as follows:

$$\begin{align*}
\{ \lambda \} & \quad w(1 - \tau) - C - I = 0 \\
\{ I \} & \quad u_h h_I = \lambda \\
\{ \tau \} & \quad u_h h_{\tau} \frac{1}{w} = \lambda \\
\{ C \} & \quad u_C = \lambda
\end{align*}$$

At the parent’s preferred point, the marginal utility of the money spent on the parent’s consumption, the parent’s monetary investments in children, and forgone earnings due to time spent with children are equated.
3.3.3 Optimal decisions if the parent migrates and child is left-behind

When the child is left-behind by a migrant parent \((M^p = 1, M^c = 0)\), the parent doesn’t have the option of spending time with the child on a regular basis: \(\tau = 0\) and \(\tau\) is no longer a decision variable. The parent’s maximization problem becomes:

\[
\max_{I,C} \{ u(C, h(I, G, 0)) + \lambda (w - C - I) \}
\]

The first order conditions are:

\[
\begin{align*}
\{\lambda\} \quad & w - C - I = 0 \\
\{I\} \quad & u_I h_I = \lambda \\
\{C\} \quad & u_C = \lambda
\end{align*}
\]

In this case, the marginal utility of the parent’s monetary investments in children is only equated with the marginal utility of money spent on the parent’s consumption.

4 Results

I derive two main results related to (i) the effect of parental migration on children’s human capital, and (ii) the effects of government policies that increase services for children. First, I show that given the model assumptions (government spending is a substitute for parental spending, and parent time with the child is weakly complementary to spending on the child), children’s human capital decreases as they become left-behind by migrant parents. Second, I show that government policies that increase services for children can have the unintended consequence of separating children from parents due to endogenous migration effects, thereby reducing child human capital. The second result is a corollary to the first. Additional details are provided below.

4.1 The impact of parental migration on children’s human capital

I now present the first main result of the paper. Children are left-behind as their parents cross a margin of indifference between living with the child versus migrating while the child stays behind. I therefore consider a family in which the parent is at such a margin of
indifference. I show that the child’s human capital will be higher if the child lives with the parent, as formalized below.

**Result 1** \( h^*_{10} < h^*_1 \) and \( h^*_{10} < h^*_{00} \) for families at the margins \( U^*_{10} = U^*_{11} \) and \( U^*_{10} = U^*_{00} \), respectively.

The intuition behind this result is as follows. Because the parent is indifferent between the two migration scenarios, one argument of her utility function (either parental consumption or child human capital) must decrease if the other increases when the child becomes left-behind. In Appendix A.2, I show that parental consumption increases and therefore child human capital decreases if the child is left-behind, rather than vice-versa. Decreasing parental consumption once the child is left-behind would not be optimal because the parent’s earning opportunities have increased: the parent lives in the higher-wage area, and can increase her labor supply because she no longer provides child care. Furthermore, the child’s human capital mechanically decreases when the child is left-behind: The loss of parental time decreases the child’s human capital directly, and also indirectly by reducing the productivity of the parent’s monetary investments in the child. Although parental remittances can partially compensate for the decrease in child human capital, full compensation would violate the premise that the parent was indifferent between the two migration scenarios.

## 4.2 Effects of policy changes on parental utility and children human capital, for a policy without migration effects

I consider two broad classes of policy changes (a) increasing government spending on children living in rural areas \((G_r)\), and (b) increasing government spending on children who are rural-urban migrants \((G_u)\). Both types of policies have been recently implemented in China [Lin et al., 2009, The Guardian, 2014, Wagstaff et al., 2009, Wall Street Journal, 2014].

Children become left-behind after their parents cross one of two margins:

\[
U^*_{10} = U^*_{11} \\
U^*_{10} = U^*_{00}
\]
To understand why parents might cross one of these margins as a result of a policy change, consider how $U_{11}^*$, $U_{00}^*$, and $U_{10}^*$ are affected by policy changes:

**Observation 1** If policy changes affect $G_u$ only, then the utility and human capital of children and parents living in the rural area are not affected by the policy. Likewise, if policy changes affect $G_r$ only, then the utility and human capital of children and parents living in the urban area are not affected by the policy. That is, $\frac{\partial U_{00}^*}{\partial G_u} = \frac{\partial U_{10}^*}{\partial G_u} = \frac{\partial U_{11}^*}{\partial G_r} = 0$ and $\frac{\partial h_{00}^*}{\partial G_u} = \frac{\partial h_{10}^*}{\partial G_u} = \frac{\partial h_{11}^*}{\partial G_r} = 0$.

**Observation 2** An increase in $G_u$ increases the utility and human capital of children and parents living in urban areas. Likewise, an increase in $G_r$ increases the utility and human capital of children and parents living in rural areas. That is, $\frac{\partial U_{00}^*}{\partial G_r} > 0$, $\frac{\partial U_{10}^*}{\partial G_r} > 0$, $\frac{\partial U_{11}^*}{\partial G_u} > 0$, $\frac{\partial h_{00}^*}{\partial G_r} > 0$, $\frac{\partial h_{10}^*}{\partial G_r} > 0$ and $\frac{\partial h_{11}^*}{\partial G_u} > 0$.

Intuitively, in this model, an increase in government spending on children can be shown as equivalent to a positive monetary transfer to parents, which parents optimally divide between themselves and their children. See Becker [1993]. Proofs of these observations are included in Appendix A.1.

By Observations 1 and 2, location-specific government spending increases the appeal of certain migration states:

$$\frac{\partial U_{11}^*}{\partial G_r} = 0; \frac{\partial U_{10}^*}{\partial G_r} > 0; \frac{\partial U_{00}^*}{\partial G_r} > 0$$

(1)

$$\frac{\partial U_{11}^*}{\partial G_u} > 0; \frac{\partial U_{10}^*}{\partial G_u} = 0; \frac{\partial U_{00}^*}{\partial G_u} = 0$$

(2)

Equations (1) and (2) imply that a policy that increased $G_u$ could reduce the number of left-behind children as parents cross the $U_{10}^* = U_{11}^*$ margin and bring their children to the urban area, whereas a policy that increased $G_r$ could either increase or decrease the number of left-behind children as parents cross the $U_{10}^* = U_{11}^*$ or $U_{10}^* = U_{00}^*$ margins.

### 4.3 Effects of policy changes on parental utility and children human capital, for a policy with migration effects

I now present the second main result of the paper. I consider the case in which a parent changes their decision about living with the child versus migrating while the child stays
behind due to an incremental policy change. In this case, the child’s human capital will be lower if the child lives apart from the parent (i.e., is left behind), as formalized below.

**Result 2** If a child becomes left-behind as a result of incremental changes in location-specific government spending, becoming left-behind will decrease the child’s human capital. If a child becomes no longer left-behind (rejoins his parents) as a result of incremental changes in location-specific government spending, rejoining his parents will increase the child’s human capital.

Result 2 is a corollary of Result 1. Result 1 states that \( h_{10}^* < h_{11}^* \) for families at the margin \( U_{10}^* = U_{11}^* \) and \( h_{10}^* < h_{00}^* \) for families at the margin \( U_{10}^* = U_{00}^* \). By continuity of \( h \), when we compare \( h_{10}^* \) with \( h_{11}^* \) or \( h_{00}^* \) for families sufficiently close to these two margins, the relationships \( h_{10}^* < h_{11}^* \) and \( h_{10}^* < h_{00}^* \) should still hold.

### 4.4 Robustness to families with multiple parents or grandparents

The results are robust to multiple changes in the composition of the family. In particular, I show in Appendix B that the model can be extended to allow for two-parent families, or to allow for a grandparent or other relative to care for left-behind children without affecting the results. In each of these extensions, the key channels underlying Result 1 remain the same: parental migration reduces parental time with the child and increases parental income. Some of the increased parental income is optimally used for parental consumption, as in the one-parent case. Then, given that parents are on the margin between two migration scenarios, the increase in parental consumption implies a decrease in child human capital.

### 5 Combining the results for policy analysis

Figure 2 depicts how changes in location-specific government spending on children can place children at risk of becoming left-behind, and illustrates the implications for children’s human capital development. The figure illustrates four general cases. The first two cases focus on children whose parents cross the \( U_{11}^* = U_{10}^* \) margin as a result of a policy change, and the second two cases focus on children whose parents cross the \( U_{00}^* = U_{10}^* \) margin as a result of a policy change.

In the first case, captured in Panel A of Figure 2, a child lives with his parent in the urban area before a policy that increases government spending on children in rural areas...
(\(G_r\)) is implemented. A small increase in \(G_r\) would not change the parent’s decision about the child’s migration status and as such would not affect the child’s human capital by Observation 1. If the increase in \(G_r\) were sufficiently large, however, the parent would reach and cross the \(U_{11}^* = U_{10}^*\) margin and ask the child to return to the rural area, which would harm the now “left-behind” child by Result 2. After becoming left-behind, the child would only benefit from any further increases in \(G_r\) by Observation 2. Therefore, a sufficiently large increase in \(G_r\) could overwhelm the harmful migration effects so that the net impact of the policy on child human capital is positive.

In the second case, depicted in Panel B of Figure 2, a child is left-behind in a rural area by her migrant parent before a policy change increasing government spending on rural children in urban areas (\(G_u\)) is implemented. As before, a small increase in \(G_u\) would not change the parent’s decision about the child’s migration status and as such would not affect the child’s human capital by Observation 1. If the increase in \(G_u\) were sufficiently large, the parent would reach and cross the \(U_{11}^* = U_{10}^*\) margin and bring the left-behind child to the urban area as a migrant, which would benefit the child by Result 2. Any further increases in \(G_u\) would further benefit the child by Observation 2.

The above examples focus on children whose parents cross the \(U_{11}^* = U_{10}^*\) margin of leaving them behind. Based on Equations (1) and (2), these examples capture the two ways a policy change can affect the number of left-behind children by this margin. However, children can also become left-behind after parents cross the \(U_{00}^* = U_{10}^*\) margin, as I now consider.

Equations (1) and (2) imply that changes to \(G_u\) cannot cause parents to cross the \(U_{00}^* = U_{10}^*\) margin but changes to \(G_r\) might. I cannot know without making further assumptions which direction parents might move across this margin as a result of an increase to \(G_r\), and therefore I consider both cases. First, a child living in the rural area with his parent might see his parent migrate and leave him behind after a sufficiently large increase in \(G_r\). Second, a left-behind child living in the rural area might see his migrant parent return to the rural area after a sufficiently large increase in \(G_r\). In both cases, the child lives in the rural area and therefore benefits from the increase in \(G_r\) by Observation 2. The changes to the child’s left-behind status would reduce or augment these benefits in these two cases, respectively, by Result 2. These impacts on child human capital are graphically depicted in Panels C and D of Figure 2.

These examples demonstrate two general points about the impact of increasing place-specific government spending on child human capital. An increase in government spending
Figure 2: Place-specific government spending can determine whether or not children are left-behind by migrant parents: implications for children’s human capital development

This figure illustrates how changes to location-specific government spending on children can change whether children are left-behind by migrant parents, with implications for child human capital development. The horizontal axis of each graph indicates the size of the increase in government spending on rural children in urban areas ($G_u$) or rural children in rural areas ($G_r$), and the vertical axis indicates child human capital. Red areas denote that the child is left-behind in the rural area while their parent migrates, gray areas denote that the child rejoins their migrant parent in the urban area, and blue areas denote that the child lives with their parent in the rural area.
on rural children in urban areas, $G_u$, has (a) direct effects that are beneficial for children and (b) indirect effects that are also beneficial for children, if children become no longer left-behind as a result of the policy. In this case, it is clear that children’s human capital can only be increased by the policy. However, although an increase in government spending on rural children in rural areas, $G_r$, also has direct effects that are beneficial for children, this policy could have harmful indirect effects if additional children become left-behind as a result of the policy. In a companion paper, I show that a recent increase in government spending in rural China was indeed associated with an increase in the number of children left-behind by migrant parents [Myerson, 2017].

6 Discussion

This paper addresses a puzzle in the empirical literature about the effects of being left-behind by migrant parents on children’s human capital. By exploiting the fact that children can only become left-behind after their parents cross a margin of indifference, I am able to identify a case in which the effect of being left-behind on children can be signed. I show that if government spending is a substitute for parental spending and parent time with the child is weakly complementary to spending on the child, then children’s well-being decreases as they become left-behind by migrant parents.

The model I use follows classic models of intergenerational mobility in a number of ways [Becker and Tomes, 1979, 1986, Loury, 1981, Solon, 2004]. First, parents decide on how much of their earnings to consume themselves vs. invest in their children. Second, parental monetary investments in child human capital have a positive influence but with decreasing marginal returns. In the literature, this is often captured using a semi-log functional form. Third, parental utility is determined by parental consumption and a child outcome, such as earnings or welfare, which is an increasing function of child human capital. (For simplicity, I abstract from the process that translates human capital into earnings or welfare, so that child human capital directly enters into the parental utility function.) Fourth, when a functional form is assumed for the parental utility in the literature, additive separability is typically assumed and I also employ this assumption.

Weinberg, 2001]. In the family economics field, there is growing interest in the channels through which parents’ education and income has causal effects on children’s skill development, and time with children is one such channel [Black and Devereux, 2011, Cunha et al., 2006, Cunha and Heckman, 2008, Cunha et al., 2010, Heckman, 2008]. Including child care time as an alternative to labor supply is also essential to capturing the key trade-offs that parents face when deciding whether or not to migrate for work without their children.

My model also draws on the literature related to labor migration and migration responses to policy. Following classic theoretical models in this field, I allow wages and generosity of government programs to vary by geographic location [Harris and Todaro, 1970, Sjaastad, 1962, Tiebout, 1956, Todaro, 1969a]. (See Lucas [1997] for a summary of empirical work on internal migration in response to policy in developing countries.) As in previous research, I find that agents’ migration decisions can be affected by changes in generosity of government programs, as in the welfare magnet hypothesis [Allard and Danziger, 2008, Berry et al., 2003, Borjas, 1999, Levine and Zimmerman, 1999, Schwartz and Sommers, 2014].

My findings extend the literature on the welfare magnet hypothesis by showing that migration responses to government programs can undermine the beneficial effects of these programs if the people making decisions about migration (in my model, parents) are not the intended beneficiaries of the policy (in my model, children). These findings are distinct from findings of previous studies indicating that sorting of heterogeneous agents across locations can undermine the observed effect of local policies and programs, because agents are not heterogeneous in my model [Epple and Romer, 1991, Lanzona, 1998, Rosenzweig and Wolpin, 1988, Todaro, 1969b, Todd, 2007].

Finally, my analysis contributes to the literature on marginal treatment effects, defined as the impact of taking a certain path of action for individuals at the margin of doing so [Carneiro et al., 2010, Heckman and Vytlacil, 1999, 2001, 2007, Heckman, 2010]. This paper demonstrates a new approach to signing difficult-to-sign comparative statics in policy analysis using marginal treatment effects. By focusing on agents who are at the margin of leaving their child behind, I obtain an additional equation that enables me to sign the impact of becoming left-behind on children.³ This approach differs from the existing literature on marginal treatment effects [Aakvik et al., 2005, Basu et al., 2007, Carneiro et al.,

³In the parlance of Heckman and Vytlacil [2007] and Carneiro et al. [2010], my Result 1 signs a marginal treatment effect (the impact of becoming left-behind on children who are at the margin of becoming left-behind), while Result 2 signs a marginal policy-relevant treatment effect (the impact of a small policy change) for the subset of families sufficiently close to a margin of leaving children behind.
Rather than estimating distributions of marginal treatment effects, I focus on broad conclusions that can be drawn about all marginal agents. A disadvantage of this approach is that I sign the quantity of interest without pinning down its magnitude or variability. However, this strategy simplifies the application of marginal treatment effects to policy-relevant questions and is also, to my knowledge, novel.

Because becoming left-behind lowers a child’s human capital, endogenous migration effects can augment or undermine the benefits of government programs that target rural children. Endogenous migration effects can only increase the benefits of government programs for rural-urban migrant children, but could undermine the benefits of an increase in government programs for children in rural areas if these programs cause more children to become left-behind. It follows that in many cases, a larger human capital benefit could be realized per RMB spent by targeting programs to rural-urban migrant children rather than rural children in rural areas.

The extent to which migration effects undermine the benefits of government spending on children in rural areas depends on two key factors: the size of the spending increase, and the number of parents near the relevant margins of migration. For example, if most parents were close to the margin of migrating with vs. without their children, migration effects could fully undermine the benefits of a small increase in government spending on rural children. The number of additional children who could become left behind as a result of a small policy change is an empirical question, and one I address in a companion paper [Myerson, 2017]. However, Observation 2 indicates that a sufficiently generous increase in spending on rural children can generate large enough benefits to ensure that the net impact on children’s human capital is positive.

References


Bingyan He, Jingyi Fan, Ni Liu, Huijuan Li, Yanjun Wang, Joshua Williams, and Kaishing


Yun Huang, Xiao-Ni Zhong, Qing-Ying Li, Dan Xu, Xuan-Lin Zhang, Chao Feng, Guo-Xiu Yang, Yun-Yun Bo, and Bing Deng. Health-related quality of life of the rural-China


Qiming Liu and Barry Reilly. Income transfers of Chinese rural migrants: some empirical


Yao Lu. The Gender-Differentiated Impact of Father Migration on Left-Behind Children Evidence from Gansu Province, China. 2012b.


Rebecca Myerson. Which children become left behind by migrant parents after a policy


Hongwei Xu and Yu Xie. The Causal Effects of Rural-to-Urban Migration on Chil-


A  Proofs

A.1  Observations

A.1.1  Proof of Observation 1

Without loss of generality, consider the example of a child who lives in an urban area ($M^c = 1$); say that $G_r$ increases due to a policy change but the change is not large enough to provoke the parent of the child to send him home. For this child, $G$ does not change because $G = G_u$, and there is no change in $G_u$.

If $G$ does not change and no other inputs relevant to parental optimization change, then $C$, $I$ and $\tau$ also do not change. Thus, without any change in any of the inputs of the parent’s decision problem, child welfare ($h(I, G, \tau)$) and parental utility $u(C, h)$ do not change.

A.1.2  Proof of Observation 2

As a first step, I show that governmental investment of level $G$ is equivalent to having the parents receive a monetary transfer of a certain amount, $k(G)$.

Adding notation to fix ideas, say that if $G$ is set to 0 but the parent is given a transfer of size $k(G)$ then the parent will spend $I^*_k(G)$ on the child. If instead the government directly invests amount $G$ in the child and gives no transfer to the parent, the parent will choose to spend $I^*_G$ on the child. I want to show that for any amount of government spending on children $G$, there exists a direct transfer to parents of amount $k(G)$ that would produce the same $h^*$ and $C^*$, thereby producing identical levels of child welfare and utility for parents.

I will work with the optimization problem for children who live in the same place as parents, but the equations I obtain will also hold for left-behind children. This is because the two first-order conditions that both equal $\lambda$ and are set to equal each other are equivalent to two first-order conditions for the left-behind child case, and the third first-order condition for the left-behind child case is unaffected by the change to the budget constraint (see section 3.3.3).

With the hypothetical transfer $k(G)$ included and $G$ eliminated, the budget constraint becomes:

$$w(1 - \tau) + k(G) = C + I_k(G)$$

and therefore the parent’s maximization problem becomes:
\[
\max_{I_k(G), \tau, C} \left\{ u(C, h(I_k(G), 0, \tau)) + \lambda \left( w(1 - \tau) + k(G) - C - I_k(G) \right) \right\}
\]

This yields the first-order conditions:

\[
\begin{align*}
\{ \lambda \} & \quad w(1 - \tau) + k(G) - C - I_k(G) = 0 \\
\{ I \} & \quad u_h I = \lambda \\
\{ \tau \} & \quad u_h \tau \frac{1}{w} = \lambda \\
\{ C \} & \quad u_C = \lambda
\end{align*}
\]

Inada conditions imply that the second and fourth first-order conditions can be combined to yield:

\[u_h h_{I_k(G)} = u_C\]

The comparable condition for the cases without transfer \( k(G) \) and where \( G > 0 \) is:

\[u_h h_G = u_C \quad \text{(3)}\]

Therefore if we define the relationship between \( G \) and \( k(G) \) so that:

\[h_G = h_{I_k(G)} \quad \text{(4)}\]

then the transfer \( k(G) \) will produce an identical parental optimization problem and identical outcomes to governmental investment in the child of amount \( G \).

I now establish \( \frac{\partial k(G)}{\partial G} > 0 \) by examining what changes are needed to maintain Equation (4) when \( G \) changes. First, I examine the left hand size of Equation (4). \( h_{I_k(G)} < 0 \) (Assumption 3) implies that a decrease in \( G \) would increase \( h_I \), i.e., increase the left-hand side of Equation (4). The right-hand side of Equation (4), \( h_{I_k(G)} \), must therefore also increase if \( G \) decreases. \( h_{II} < 0 \) (Assumption 3) implies that increasing \( h_{I_k(G)} \) requires decreasing \( I \). Decreasing \( I \) can be accomplished by decreasing the parent’s budget - that is, by decreasing the transfer to parents \( k(G) \). In summary, if Equation (4) holds, a decrease in \( G \) implies an decrease in \( k(G) \). By similar logic, an increase in \( G \) would imply an increase in \( k(G) \).

I conclude that in terms of impacts on parental utility and children’s welfare, an increase
in $G$ produces an effect equivalent to a monetary transfer to parents, i.e., an increase in $k(G)$. The finding that government spending on children can be equivalent to a monetary transfer to parents is also discussed in Becker’s Treatise on the Family (Becker, 1993, p.192).

Having established that an increase in $G$ is equivalent to a transfer of money to parents of amount $k(G)$, I next consider how $h^*$ changes when parents receive such a transfer. By Assumption 1, parental utility is increasing in both consumption $C$ and human capital of children $h$, but with decreasing marginal returns to each. As a result, the transfer $k(G)$ must be split so that both $C^*$ and $h^*$ increase, because to do otherwise would violate the first-order conditions as combined in Equation (3). In summary:

$$\frac{\partial C^*}{\partial k(G)} > 0 \text{ and } \frac{\partial k(G)}{\partial G} > 0 \implies \frac{\partial C^*}{\partial G} > 0$$

$$\frac{\partial h^*}{\partial k(G)} > 0 \text{ and } \frac{\partial k(G)}{\partial G} > 0 \implies \frac{\partial h^*}{\partial G} > 0$$

I have shown that $h^*$ is increasing in $G$ and $u(C^*, h^*)$ is increasing in $G$. This is Observation 2.

### A.2 Proof of Result 1

#### A.2.1 First margin: $U_{10}^* = U_{11}^*$

I first consider the $U_{10}^* = U_{11}^*$ margin, a situation in which a migrating parent is just indifferent between bringing their child to the urban area or leaving the child behind. For parents at this margin, three cases exist:

1. $C_{10}^* = C_{11}^*$ and $h_{10}^* = h_{11}^*$
2. $C_{10}^* < C_{11}^*$ and $h_{10}^* > h_{11}^*$
3. $C_{10}^* > C_{11}^*$ and $h_{10}^* < h_{11}^*$

These are the only three possible cases because utility takes the form $U = u(C, h)$ where $u_C > 0$ and $u_h > 0$ (parents care about children and also enjoy consumption). Therefore, a case such as $C_{10}^* < C_{11}^*$ and $h_{10}^* < h_{11}^*$ would violate $U_{10}^* = U_{11}^*$.

**Scenario 1, $C_{10}^* = C_{11}^*$ and $h_{10}^* = h_{11}^*$, can be ruled out.** First, I can prove by contradiction that in this scenario, it must be that $\tau_{11}^* > 0$ and therefore $I_{10}^* > I_{11}^*$. If $C_{10}^* = C_{11}^*$ and $\tau_{11}^* = 0$, this would yield $I_{10}^* = I_{11}^*$ by the budget constraint, then by $G_r > G_u$ (assumption 5) this yields $h_{10}^* > h_{11}^*$, which contradicts $U_{10}^*(C_{10}^*, h_{10}^*) = U_{11}^*(C_{11}^*, h_{11}^*)$. Therefore, marginal...
families must have $\tau_{11}^* > 0$ meaning that if the child is left-behind, parental income will be higher by $\tau_{11}^* w_u$. By $C_{10}^* = C_{11}^*$ and the budget constraint, this implies $I_{10}^* > I_{11}^*$.

I now show that $I_{10}^* > I_{11}^*$, $C_{10}^* = C_{11}^*$, $\tau_{11}^* > 0$, and $h_{10}^* = h_{11}^*$ would violate the first-order conditions. I first apply the assumptions $h_{II} < 0$ (assumption 3), $h_{I\tau} \geq 0$ (assumption 4), $G_r > G_u$ (assumption 5) and $h_{IG} < 0$ (assumption 3 again) to obtain:

$$h_I(I_{10}^*, G_r, 0) < h_I(I_{11}^*, G_u, \tau_{11}^*)$$  (5)

According to the first order conditions in sections 3.3.2 and 3.3.3, the optimal decisions if children are left-behind or migrate with their parents must satisfy, respectively:

$$u_h(C_{10}^*, h_{10}^*) h_I(I_{10}^*, G_r, 0) = u_C(C_{10}^*, h_{10}^*)$$  (6)

$$u_h(C_{11}^*, h_{11}^*) h_I(I_{11}^*, G_u, \tau_{11}^*) = u_C(C_{11}^*, h_{11}^*)$$  (7)

I start with equation (6). Replacing using $C_{11}^* = C_{10}^*$ and $h_{11}^* = h_{10}^*$, I obtain:

$$u_h(C_{11}^*, h_{11}^*) h_I(I_{10}^*, G_r, 0) = u_C(C_{11}^*, h_{11}^*)$$

Then substituting using equation (5), I obtain the inequality:

$$u_h(C_{11}^*, h_{11}^*) h_I(I_{11}^*, G_u, \tau_{11}^*) > u_C(C_{11}^*, h_{11}^*)$$

which violates equation (7), one of the first order conditions.

I conclude that this scenario is not possible, because it would involve taking optimal decisions that violate the first order conditions and would therefore not be optimal. That is, I can rule out $h_{10}^* = h_{11}^*$ for families at the margin.

**Scenario 2, $C_{10}^* < C_{11}^*$ and $h_{10}^* > h_{11}^*$, can be ruled out.** This case can be also ruled out by a similar violation of the same first order conditions.

First, $C_{10}^* < C_{11}^*$ implies $I_{10}^* > I_{11}^*$. Then $I_{10}^* > I_{11}^*$ plus the assumptions 3, 4, and 5 imply equation (5) as in the previous scenario.

Before examining the first-order conditions, I establish a few more properties. I had assumed that $u(C, h) = \mu(C) + \nu(h)$, $u_{hh} < 0$, and $u_{CC} < 0$ (assumptions 1 and 2). If these
properties hold, $C^*_{10} < C^*_{11}$ would imply:

$$\mu_C (C^*_{11}) < \mu_C (C^*_{10}) \quad (8)$$

Likewise, $h^*_{10} > h^*_{11}$ would imply:

$$v_h (h^*_{11}) > v_h (h^*_{10}) \quad (9)$$

I then work with the first-order condition for left-behind children:

$$u_h (C^*_{10}, h^*_{10}) h_I (I^*_{10}, G_r, 0) = u_C (C^*_{10}, h^*_{10})$$

Applying additive separability of the utility function yields:

$$v_h (h^*_{10}) h_I (I^*_{10}, G_r, 0) = \mu_C (C^*_{10})$$

But equations (8) and (9) imply:

$$v_h (h^*_{11}) h_I (I^*_{10}, G_r, 0) > \mu_C (C^*_{11})$$

Finally, applying equation (5) and additive separability of the utility function again yields the final inequality:

$$u_h (C^*_{11}, h^*_{11}) h_I (I^*_{11}, G_u, \tau^*_{11}) > u_C (C^*_{11}, h^*_{11})$$

which would violate the first-order conditions (equation (7)).

I conclude that this scenario is also not possible, because the optimal decisions would violate the first order conditions and would therefore not be optimal.

**Scenario 3, $C^*_{10} > C^*_{11}$ and $h^*_{10} < h^*_{11}$, must hold.** Having shown that $C^*_{10} \neq C^*_{11}$ and $C^*_{10} \not< C^*_{11}$, I conclude that $C^*_{10} > C^*_{11}$. In order to satisfy $U^*_{10} = U^*_{11}$, $C^*_{10} > C^*_{11}$ implies $h^*_{10} < h^*_{11}$.

**A.2.2 Second margin: $U^*_{10} = U^*_{00}$**

The proof for the $U^*_{10} = U^*_{00}$ margin, the situation in which a parent is just indifferent between living with their child in the rural area vs. migrating and leaving the child behind,
proceeds along similar lines. Three cases exist at the $U_{10}^* = U_{00}^*$ margin:

1. $C_{10}^* = C_{00}^*$ and $h_{10}^* = h_{00}^*$
2. $C_{10}^* < C_{00}^*$ and $h_{10}^* > h_{00}^*$
3. $C_{10}^* > C_{00}^*$ and $h_{10}^* < h_{00}^*$

In this case, the higher wages in the left-behind child scenario ($w_u > w_r$ by assumption 6) translates to higher income for the parent regardless of the value of $\tau_{00}^*$ (i.e., $w_u > w_r (1 - \tau_{00}^*)$ for any $0 \leq \tau_{00}^* \leq 1$).

**Scenario 1, $C_{10}^* = C_{00}^*$ and $h_{10}^* = h_{00}^*$, can be ruled out.** Due to the increase in parental earnings increase and $C_{10}^* = C_{00}^*$, it must be that $I_{10}^* > I_{00}^*$ by the budget constraint.

$I_{10}^* > I_{00}^*$, $C_{10}^* = C_{00}^*$, $\tau_{00}^* \geq 0$, and $h_{10}^* = h_{00}^*$ would violate the first-order conditions. To show this, I first apply the assumptions $h_{II} < 0$ (assumption 3) and $h_{I\tau} \geq 0$ (assumption 4) to obtain:

$$h_I(I_{10}^*, G_r, 0) < h_I(I_{00}^*, G_r, \tau_{00}^*) \quad (10)$$

The optimal decisions if children are left-behind or migrate with their parents must satisfy the following first-order conditions, respectively:

$$u_h(C_{10}^*, h_{10}^*) h_I(I_{10}^*, G_r, 0) = u_C(C_{10}^*, h_{10}^*) \quad (11)$$

$$u_h(C_{00}^*, h_{00}^*) h_I(I_{00}^*, G_r, \tau_{00}^*) = u_C(C_{00}^*, h_{00}^*) \quad (12)$$

I start with equation (11). Replacing using $C_{00}^* = C_{10}^*$ and $h_{00}^* = h_{10}^*$, I obtain:

$$u_h(C_{00}^*, h_{00}^*) h_I(I_{10}^*, G_r, 0) = u_C(C_{00}^*, h_{00}^*)$$

Then substituting using equation (10), I obtain the inequality:

$$u_h(C_{00}^*, h_{00}^*) h_I(I_{00}^*, G_r, \tau_{00}^*) > u_C(C_{00}^*, h_{00}^*)$$

which violates equation (12).

I conclude that this scenario is not possible, because it would involve taking optimal decisions that violate the first order conditions and would therefore not be optimal. That is, I can rule out $h_{10}^* = h_{00}^*$ for families at the $U_{10}^* = U_{00}^*$ margin.
Scenario 2, \( C_{10}^* < C_{00}^* \) and \( h_{10}^* > h_{00}^* \), can be ruled out. This case can be also ruled out by a similar violation of the same first order conditions.

First, \( C_{10}^* < C_{00}^* \) implies \( I_{10}^* > I_{00}^* \). Then \( I_{10}^* > I_{00}^* \) plus assumptions 3 and 4 imply equation (10) as in the previous scenario.

Also, \( C_{10}^* < C_{00}^* \) and \( h_{10}^* > h_{00}^* \) imply:

\[
\mu_C(C_{00}^*) < \mu_C(C_{10}^*) \quad (13)
\]

\[
\nu_h(h_{00}^*) > \nu_h(h_{10}^*) \quad (14)
\]

As before, I examine the first-order conditions by starting with a first-order condition for left-behind children, equation (11). Applying additive separability of the utility function, and equations (13) and (14), I obtain:

\[
\nu_h(h_{00}^*) h_I(I_{00}^*, G_r, 0) > \mu_C(C_{00}^*)
\]

Finally, applying equation (10) and additive separability of the utility function again yields the final inequality:

\[
u_h(C_{00}^*, h_{00}^*) h_I(I_{00}^*, G_r, \tau_{00}^*) > u_C(C_{00}^*, h_{00}^*)
\]

which would violate the first-order conditions (equation (12)).

I conclude that this scenario is also not possible, because the optimal decisions would violate the first order conditions and would therefore not be optimal.

Scenario 3, \( C_{10}^* > C_{00}^* \) and \( h_{10}^* < h_{00}^* \), must hold. Having shown that \( C_{10}^* \neq C_{00}^* \) and \( C_{10}^* \neq C_{00}^* \), I conclude that \( C_{10}^* > C_{00}^* \). In order to satisfy \( U_{10}^* = U_{00}^* \), \( C_{10}^* > C_{00}^* \) implies \( h_{10}^* < h_{00}^* \).

In summary, I have shown that \( h_{10}^* < h_{11}^* \) for parents at \( U_{10}^* = U_{11}^* \) and \( h_{00}^* < h_{00}^* \) for parents at \( U_{10}^* = U_{00}^* \). This is Result 1.

B Extensions of the model

In this section, I consider two extensions to the model: adding grandparents who can care for the child in the rural area when the parent migrates, and allowing families to have two
parents rather than one parent. I add new notation and map back to the proofs above to show that Results 1 and Result 2 still hold when one or more parents migrate and leave their child behind.

B.1 Grandparents care for left-behind children

First, I consider the case where grandparents living in the rural area help take care of children who are left-behind by migrant parents. In this case, the relevant time input to the child’s welfare function is \( \tilde{\tau} \), total quality-adjusted child care time from both parents and grandparents. I define \( \tilde{\tau} \) as \( \tilde{\tau} = \tau_0 + \tau \), where \( \tau \) captures parental time with the child and \( \tau_0 \) captures grandparent time with the child. With this adjustment to the notation, parents’ choices and the corresponding utility are as follows:

\[
U = \max_{M^p, M^c} \left\{ \begin{array}{ll}
U_{11}^* &= u(C_{11}^*, I_{11}, G_u, \tau_{11}^*) \\
U_{00}^* &= u(C_{00}^*, I_{00}, G_r, \tau_0 + \tau_{00}^*) \\
U_{10}^* &= u(C_{10}^*, I_{10}, G_r, \tau_0) \\
\end{array} \right.
\]

Recall that Result 1 focuses on parents who are indifferent between leaving their child behind vs. another migration scenario (that is, parents at the \( U_{10}^* = U_{11}^* \) or \( U_{10}^* = U_{00}^* \) margin). In the case with grandparents, these two margins can be denoted, respectively:

\[
\begin{align*}
    u(C_{10}^*, I_{10}, G_r, \tau_0) &= u(C_{11}^*, I_{11}, G_u, \tau_{11}^*) \\
    u(C_{10}^*, I_{10}, G_r, \tau_0) &= u(C_{00}^*, I_{00}, G_r, \tau_0 + \tau_{00}^*) 
\end{align*}
\]

I first consider Equation (15) (the \( U_{10}^* = U_{11}^* \) case with grandparents). This equation is identical to the case without grandparents except that \( \tau_0 \) takes the place of 0 on the left-hand side. The \( \tau_{11}^* \geq 0 \) argument in the original proof would therefore be replaced with \( \tau_{11}^* \geq \tau_0 \): if \( \tau_{11}^* \geq \tau_0 \), the proof proceeds as before.

To show that \( \tau_{11}^* \geq \tau_0 \), I use proof by contradiction. If \( \tau_{11}^* < \tau_0 \), it would follow that leaving the child behind dominates migrating with the child (i.e., \( U_{10}^* > U_{11}^* \)), violating the premise \( U_{10}^* = U_{11}^* \). In particular, if the migrant parent sent the child to live in the rural area then: (a) the parent’s income would weakly increase (\( w_u \geq w_u (1 - \tau_{11}^*) \)), producing increases in both \( C^* \) and \( I^* \); \(^4\) (b) the child would receive a higher government transfer \( (G_r > G_u) \); and (c) the child would benefit more from time with family \( (\tau_0 > \tau_{11}^*) \). Mechanically,

\[^4\text{As argued before, increasing only } I \text{ or only } C \text{ would violate the first-order conditions.}\]
this yields $u\left(C_{10}^*, h\left(I_{10}^*, G_r, \tau_0\right)\right) > u\left(C_{11}^*, h\left(I_{11}^*, G_u, \tau_{11}^*\right)\right)$ or $U_{10}^* > U_{11}^*$, which violates the premise of $U_{10}^* = U_{11}^*$. Based on this contradiction, I conclude that $\tau_{11}^* \geq \tau_0$ holds and the proof follows as in the $U_{10}^* = U_{11}^*$ case without grandparents as argued above.

I next consider Equation (16) ($U_{10}^* = U_{00}^*$). This equation is identical to the case without grandparents except that $\tau_0$ is included on both the right- and left-hand side. The $\tau_{00}^* \geq 0$ argument in the proof is therefore replaced with $\tau_{00}^* + \tau_0 \geq \tau_0$. Because $\tau_{00}^* + \tau_0 \geq \tau_0$ simplifies to $\tau_{00}^* \geq 0$, the proof follows exactly as in the $U_{10}^* = U_{00}^*$ case without grandparents.

However, it could be argued that grandparents would spend less time on child care when the parent also lives in the rural area. In this case, Equation (16) would become

$$u\left(C_{10}^*, h\left(I_{10}^*, G_r, \tau_0\right)\right) = u\left(C_{00}^*, h\left(I_{00}^*, G_r, \tau_0' + \tau_{00}^*\right)\right)$$

where $\tau_0' < \tau_0$. As in the $U_{10}^* = U_{11}^*$ case with grandparents, I use proof by contradiction to show that quality-adjusted child-care time provided by the family must be greater when the child and parent live together than when the child is left behind (that is, $\tau_{00}^* + \tau_0' \geq \tau_0$). If we suppose to the contrary that $\tau_{00}^* + \tau_0' < \tau_0$, then leaving the child behind would dominate staying with the child ($U_{10}^* > U_{00}^*$), violating the premise of $U_{10}^* = U_{00}^*$. In particular, if the parent migrated and left the child behind, then (a) the parent’s income would increase ($w_u > w_r \left(1 - \tau_{00}^*\right)$) producing increases in $I^*$ and $C^*$; (b) the child would receive the same transfer from the government ($G_r$); and (c) the child would benefit from an increase in quality-adjusted time with family ($\tau_0 > \tau_{00}^* + \tau_0'$). These benefits to both parent and child would yield $U_{10}^* > U_{00}^*$. Based on this contradiction, I conclude that total quality-adjusted child care provided by the family must decline when the child is left-behind. The proof then follows as in the $U_{10}^* = U_{00}^*$ case without grandparents after replacing the $\tau_{11}^* \geq 0$ argument in the original proof with $\tau_{00}^* + \tau_0' \geq \tau_0$.

### B.2 Two-parent family

If one or more parents decide to migrate without bringing the child, this would increase the household’s total earning power while limiting total parental time with the child, the same trade-offs analyzed in the original setup.

I assume the two parents (Parent A and Parent B) work together to act as a single decision-maker with the same preferences as the single-parent case. That is, the parents care about their child and their own consumption with diminishing marginal returns to each, and their utility is additively separable across these two inputs (Assumptions 1 and 2...
still hold). Either parent could earn wages \( w_u \) in the urban areas and \( w_r \) in the rural areas where \( w_u > w_r \) (Assumption 6), and time with either parent is equally productive for the child. To reduce notation without loss of generality, let Parent B denote the parent who stays home with the child if only one parent migrates.

The parents’ options related to family members’ migration can then be summarized as follows:

- \( M^A = 1, M^B = 1, \) and \( M^c = 1 \) (both parents migrate; child accompanies parents)
- \( M^A = 1, M^B = 1, \) and \( M^c = 0 \) (both parents migrate and child is left-behind alone)
- \( M^A = 1, M^B = 0, \) and \( M^c = 0 \) (one parent migrates and child is left-behind with the other parent)
- \( M^A = 0, M^B = 0, \) and \( M^c = 0 \) (no family members migrate)

I indicate parents’ optimal decisions in each of these migration scenarios by extending the notation used in the single-parent case. For example, \( C^*_{111} \) denotes the parents’ optimal consumption when \( M^A = 1, M^B = 0, \) and \( M^c = 0 \) (that is, the first subscript in \( C^*_{100} \) indicates \( M^A = 1 \), the second indicates \( M^B = 0 \), and the third indicates \( M^c = 0 \)). Similarly, \( U^*_{100} \) is the utility of the parents at the optimum when \( M^A = 1, M^B = 0, \) and \( M^c = 0 \), and \( \tau^*_{100} \) denotes parental time with the child at the optimum in this scenario. (In this example, child care time \( \tau^*_{100} \) is provided solely by the parent who lives with the child, Parent B.) The optimal choices and utility for the other migration scenarios are similarly denoted using stars and subscripts.

With this notation, we can summarize parental choices and the corresponding utility as follows:

\[
U = \max_{M^A, M^B, M^c} \begin{cases} 
U^*_{111} = u(C^*_{111}, h(I^*_{111}, G_u, \tau^*_{111})) \\
U^*_{110} = u(C^*_{110}, h(I^*_{110}, G_r, 0)) \\
U^*_{100} = u(C^*_{100}, h(I^*_{100}, G_r, \tau^*_{100})) \\
U^*_{000} = u(C^*_{000}, h(I^*_{000}, G_r, \tau^*_{000})) 
\end{cases}
\]

There are now several margins relevant to leaving a child behind:

\[
\begin{align*}
U^*_{110} &= U^*_{111} \\
U^*_{100} &= U^*_{000} \\
U^*_{110} &= U^*_{100} \\
U^*_{100} &= U^*_{000}
\end{align*}
\]
In each case, the left-hand side indicates the case where the child is left-behind according to the definitions used in national surveys, that is, the child lives at home in the rural area while one or more parents migrate away [All China Women’s Federation, 2008, 2013]. (For example, in the $U_{110}^* = U_{000}^*$ case, the parents are indifferent between both migrating and leaving the child behind vs. not migrating.) I now consider each of these margins in turn and show that the results derived in the one-parent case still hold: the migration of one or more parents reduces child welfare.

At margins where both parents choose to locate together as a single unit, the proofs follow as in the one-parent case. The proof for $U_{110}^* = U_{111}^*$ is the same as the proof for the one-parent $U_{10}^* = U_{11}^*$ case after the following mapping of notation: the statement $\tau_{11}^* \geq 0$ from the one-parent case is replaced with $\tau_{111}^* \geq 0$. (Following the notation introduced above, $\tau_{111}^*$ represents the total time investment in the child by the two parents when all three live in the urban area). The proof then proceeds as in the one-parent case. Likewise, the proof for the $U_{110}^* = U_{000}^*$ case is the same as the one-parent $U_{10}^* = U_{00}^*$ case after mapping of notation in two parts of the proof. First, the statement $\tau_{00}^* \geq 0$ from the one-parent case is replaced with $\tau_{000}^* \geq 0$. Second, the total income of the parents is altered to accommodate the fact that there are two parents: $w_u > w_r (1 - \tau_{000}^*)$ is replaced with $2w_u > w_r (2 - \tau_{000}^*)$. (The inequality holds because $w_u > w_r$ by Assumption 6 and $\tau_{000}^* \geq 0$ by definition.) The proof then proceeds as in the one-parent case.

The result also holds for parents at the $U_{110}^* = U_{100}^*$ margin, i.e., the case where one parent has migrated and the parents are indifferent about whether or not the other parent should also migrate while the child remains in the rural area. Intuitively, parents at the $U_{110}^* = U_{100}^*$ margin face the same trade-offs as those cited in the proof for the one-parent $U_{10}^* = U_{00}^*$ case. First, the migration of the remaining parent would weakly reduce parental time with the child by $\tau_{100}^* \geq 0$. Second, the migration of the remaining parent would increase the parents’ income: $2w_u > w_r (1 - \tau_{000}^*)$ is replaced with $2w_u > w_r (2 - \tau_{000}^*)$. (The inequality holds because $w_u > w_r$ by Assumption 6 and $\tau_{000}^* \geq 0$ by definition.) The proof then proceeds as in the one-parent case.

To summarize, I have shown that the proofs of Result 1 (and by extension, Result 2) hold for two-parent families when parents are indifferent between leaving the child behind without a parental caregiver and any other migration scenario. In each case, leaving the child behind without a parental caregiver reduces the child’s welfare.

I now turn to the case where parents are indifferent between the no-migration scenario

---

5 The child remains in the rural area either way so that $G$ is unaffected.
and a scenario in which only one parent migrates. This is the $U_{100}^* = U_{000}^*$ margin. I begin by comparing the two scenarios. The child receives the same government transfer, $G_r$, in both scenarios. The migration of one parent results in higher parental income if and only if $w_u + w_r (1 - \tau_{100}^*) > w_r (2 - \tau_{000}^*)$, which simplifies to:

$$w_u - w_r > w_r (\tau_{100}^* - \tau_{000}^*)$$

Because the only constraints on $\tau_{000}^*$ and $\tau_{100}^*$ are $0 \leq \tau_{000}^* \leq 2$ and $0 \leq \tau_{100}^* \leq 1$, respectively, it is not immediately clear whether or not Inequality (17) holds. For the same reasons, it is unclear whether or not total parental time with the child decreases when one parent migrates ($\tau_{100}^* - \tau_{000}^* < 0$). Therefore, I proceed by considering cases.

Three possibilities exist for the impact of single-parent migration on parental time with the child and total parental income. First, total parental time with the child could drop after a parent migrates (i.e., $\tau_{100}^* < \tau_{000}^*$). This would increase parental income: $w_u > w_r$ plus $\tau_{100}^* < \tau_{000}^*$ imply that Inequality (17) holds. As such, the parents face the same trade-offs inherent in the one-parent $U_{10}^* = U_{11}^*$ case: when a parent migrates, parental time with the child decreases but parental income increases. The proof then follows as in the one-parent $U_{10}^* = U_{00}^*$ case, which builds on these trade-offs.

Two additional possibilities exist but can be ruled out. If parental time with the child and parental income both increased when one parent migrated (i.e., $\tau_{100}^* \geq \tau_{000}^*$ and Inequality (17) holds), the one-parent migration scenario would dominate the no-migration scenario ($U_{100}^* > U_{000}^*$). This would violate the premise that parents are indifferent between the scenarios ($U_{100}^* = U_{000}^*$). Alternatively, parental time with the child could be higher and income could be lower when one parent migrates (i.e., $\tau_{100}^* \geq \tau_{000}^*$ and Inequality (17) does not hold). This possibility can be ruled out by revealed preference. Working less, having less income, and spending more time with the child was an available but not chosen option in no-migration scenario and therefore must provide less utility than $U_{000}^*$, the utility at the optimum in the no-migration scenario. This yields $U_{100}^* < U_{000}^*$, which again violates the premise of $U_{100}^* = U_{000}^*$.

I conclude that for parents at the margin of $U_{100}^* = U_{000}^*$, the migration of one parent would result in an increase in parental income and decrease in parental time with the child. This matches the one-parent $U_{10}^* = U_{00}^*$ case, and the proof follows accordingly. Therefore, the migration of one parent should also reduce child welfare.